HEAT EXCHANGE IN THE FLOW OF RHEOLOGICALLY COMPLEX MEDIA IN A PLANE-PARALLEL CHANNEL

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A qualitative investigation is made of the problem of the nonisothermal flow of viscoplastic and "power-law" media with allowance for the dissipation of mechanical energy upon a discontinuous change in the limiting shear stress and coefficient of consistency with temperature.

Let us consider the nonisothermal flow of a rheologically complex fluid in a plane-parallel channel with allowance for dissipative heat release and for the temperature dependence of the rheological characteristics. We assume that the motion of the medium and the process of heat exchange are stabilized and that the outer surfaces of the channel walls exchange heat with the surrounding medium by Newton's law. A number of reports [1-6] have been devoted to the investigation of this problem, but heat exchange in the motion of viscoplastic materials has been inadequately studied in the literature, and the case of a sharp change in the fluidity of a medium in a relatively small temperature range, which occurs with phase transitions, stratified systems, etc., for example, also has not been considered.

The heat exchange and motion of such fluids in the presence of a dissipative factor have not been analyzed. A sharp change in fluidity is possible during the flow of these media in channels. In this case several velocity and temperature distributions can exist in the channel. The corresponding boundary problem has a nonunique solution.

The mathematical formulation of the problem is

$$\left.\begin{aligned}
&\frac{d}{dy}\left(\lambda_f \frac{dT_f}{dy}\right) + \tau^2\varphi(\tau,\ T_f) = 0, \quad 0 \leqslant y < h,\\
&\frac{d}{dy}\left(\lambda_w \frac{dT_w}{dy}\right) = 0, \qquad h \leqslant y \leqslant h+b,\\
&T_w = T_f \text{ and } \lambda_w \frac{dT_w}{dy} = \lambda_f \frac{dT_f}{dy} \quad \text{at} \quad y = h,\\
&\frac{dT}{dy} = 0 \quad \text{at} \quad y = 0;\ \lambda_w \frac{dT_w}{dy} = \alpha(T_a - T_w) \quad \text{at} \quad y = h+b.
\end{aligned}\right\} \tag{1}$$

In the flow of viscoplastic media in channels, nonuniformity of the temperature field develops owing to dissipative heat release, and under certain conditions a sharp change in the fluidity of the medium is possible. Thus, the problem under consideration can have a nonunique solution. Let us consider this "ambiguous" situation on two simple examples.

1. The motion in a plane channel of a viscoplastic medium with the rheological equation

$$\varphi(\tau,\ T) = \begin{cases} \dfrac{\tau-\tau_0}{\mu\tau} & T \leqslant T_v,\\[2mm] \dfrac{1}{\mu} & T > T_v, \end{cases}$$

where T_v is the temperature at which the plastic properties of the material disappear.

We introduce the following dimensionless variables and complexes:

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$$\Theta=\frac{T-T_v}{T_v},\quad \eta=\frac{y}{h}=\frac{\tau}{\tau_w},\quad \beta=\frac{h^2\tau_w^2}{\lambda_f\mu T_v},\ \delta=\frac{b}{h},$$

$$\mathrm{Bi}=\frac{\alpha h}{\lambda_w},\quad \lambda=\frac{\lambda_f}{\lambda_w},\quad l=\frac{3}{4}\left[1+\lambda\left(\sigma+\frac{1}{\mathrm{Bi}}\right)\right].$$

Since $\Theta(\eta)$ is a monotonically decreasing function, there exists a single value $\eta = \eta_*$ at which $\Theta(\eta_*) = 0$. Different sequences of modes of flow in the channel are possible, depending on the relationship of the quantities $\eta_0 = \tau_0/\tau_w$, η_*, 1, the temperature Θ_c of the center of the stream. We can write the value of the ambient temperature Θ_a for each of them. If $\Theta_c > 0$:

1) $\eta_0 \leqslant \eta_* \leqslant 1$ (Newtonian flow for $\eta \leqslant \eta_*$ and shear viscoplastic flow for $\eta_* < \eta \leqslant 1$)

$$\Theta_a=-\frac{\beta}{12}[-3-\eta_*^4+4\eta_0(1-\eta_*^3)]-\frac{2}{9}\beta l[2-3\eta_0(1-\eta_*^2)],\ \Theta_c=\frac{\beta\eta_*^4}{12}.$$

2) $\eta_* \leqslant \eta_0 \leqslant 1$ (Newtonian $\eta < \eta_*$, quasi-Newtonian $\eta_* \leqslant \eta \leqslant \eta_0$, and viscoplastic flow $\eta_0 < \eta \leqslant 1$)

$$\Theta_a=-\frac{\beta}{12}[-3-\eta_0^4+4\eta_0-4\eta_*^4]-\frac{2}{9}\beta l[2-3\eta_0-\eta_0^3+2\eta_*^3],\quad \Theta_c=\frac{\beta\eta_*^4}{12}.$$

3) $\eta_* \leqslant 1 \leqslant \eta_0$ (Newtonian flow $\eta \leqslant \eta_*$, and a quasisolid core at the wall $\eta_* < \eta \leqslant 1$)

$$\Theta_a=-\frac{\beta}{3}\eta_*^4\left(\frac{4}{3}\frac{l}{\eta_*}-1\right),\ \Theta_c=\frac{\beta\eta_*^4}{12}.$$

4) $\eta_* > 1$ (a Newtonian fluid flows in the channel)

$$\Theta_a=-\frac{\beta}{4}\left(\frac{16}{9}l-1\right)+\Theta_c$$

If $\Theta_c < 0$:

5) $\eta_0 < 1$ (quasisolid $\eta \leqslant \eta_0$ and and quasiplastic $\eta_0 \leqslant 1$ zones)

$$\Theta_a=-\frac{\beta}{12}[-3+4\eta_0-\eta_0^4]-\frac{2}{9}\beta l(1-\eta_0)^2(2+\eta_0)+\Theta_c.$$

6) $\eta_0 \geqslant 1$ (a quasisolid core fills the channel)

$$\Theta_a=\Theta_c.$$

Graphs of the function $\Theta_a(\Theta_c)$ are shown in Figs. 1 and 2. The minimum value of Θ_a for $\Theta_c > 0$ shifts to the left with an increase in the limiting shear stress η_0. This is explained by the fact that an increase in the size of the quasi-solid zone of the stream η_0 leads to a decrease in the total dissipation of mechanical energy Φ_Σ in the fluid stream. But from (1) we have

$$T_a=T|_{y=h+b}-\frac{\Phi_\Sigma}{\alpha}. \tag{2}$$

Consequently, the smaller Φ_Σ, the faster $|T|_{y=h+b}-\Phi_\Sigma/\alpha|$ reaches the maximum value and the temperature curve $\Theta_a(\Theta_c)$ reaches a minumum ($\Theta_c > 0$).

As seen from Figs. 1 and 2, when $\Theta_a \leqslant \Theta_a^m$ and $\Theta_a \geqslant \Theta_a^o$ the boundary problem (1) has a unique solution, while when $\Theta_a^m < \Theta_a < \Theta_a^o$ it has three solutions, with one of them (when $\partial\Theta_a/\partial\Theta_c < 0$) being unstable. It follows from (2) $(\partial T|_{y=h+b}/\partial T_c) < (1/\alpha)(d\Phi_\Sigma/dT_c)$ when $\partial T_a/\partial T_c < 0$, i.e., with an increase in the temperature of the center of the channel the dissipative heat release increases more than the heat transfer to the ambient medium and the fluid abruptly heats up. Thus, a mode of flow in which $\partial T_a/\partial T_c < 0$ does not occur.

In actual situations a discontinuous transition M → N (Fig. 1a) occurs at $\Theta_a = \Theta_a^o$. Let us determine what mode is established after the heat-up. Since $\Theta_a|_{\Theta_c=0}-\Theta_a|_{\Theta_c=\beta\eta_0^4/12} > 0$, viscoplastic flow cannot change into the three-zone mode 2) with an increase in Θ_a. Let us study the difference

$$\Delta\Theta_a(\eta_0)=\Theta_a|_{\Theta_c=0}-\Theta_a|_{\Theta_c=\beta/12}=\frac{\beta}{12}\left[1+4\eta_0-\eta_0^4+\frac{8}{3}l\eta(\eta_0^3-3)\right].$$

Since $\Delta\Theta_a(0) > 0$, $\Delta\Theta_a(1) < 0$, and $d[\Delta\Theta_a(\eta_0)]/d\eta_0 < 0$ $(0 \leq \eta_0 \leq 1)$, in the segment [0, 1] the function $\Delta\Theta_a(\eta_0)$ has a single value $\eta_0 = \bar{\eta}_0$ at which $\Delta\Theta_a(\eta_0) = 0$. The quantity $\bar{\eta}_0$ is esti-

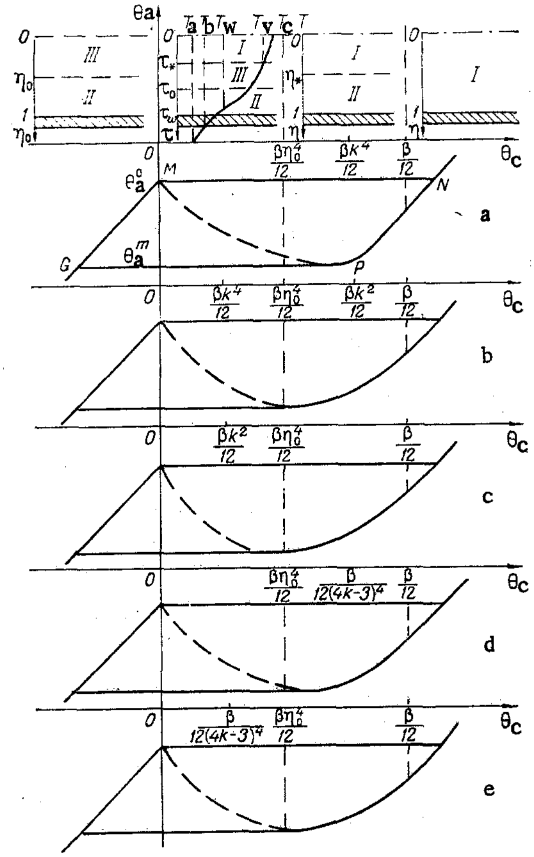

Fig. 1

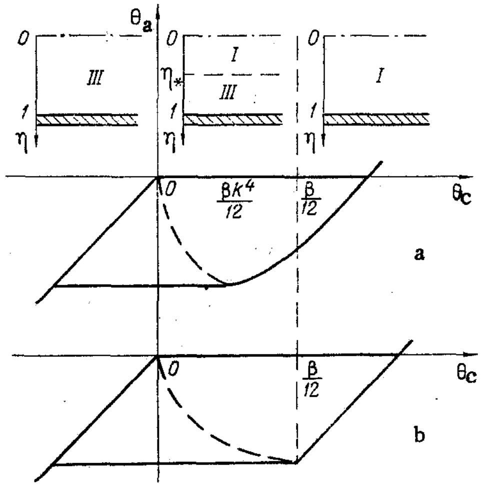

Fig. 2

Fig. 1. Dependence $\Theta_a(\Theta_c)$ for a) $\eta_0 < k < 1$; b) $k < \eta_0 < \sqrt{k} < 1$; c) $\sqrt{k} < \eta_0 < 1$; d) $\eta_0 < 1/(4k-3) < 1$; e) $1/(4k-3) < \eta_0 < 1$ (I: zone of Newtonian flow; II: zone of viscoplastic flow; III: quasisolid zone; same in Figs. 2 and 3).

Fig. 2. Dependence $\Theta_a(\Theta_c)$ for a) $k < 1 < \eta_0$; b) $1/(4k-3) < 1 < \eta_0$.

mated as follows:

$$\bar{\eta}_0 \approx 1-\left[2\left(\frac{4}{3}l-1\right)\right]^{\frac{1}{3}}, \quad \text{if } l \gtrsim 3/4, \quad \bar{\eta}_0 \approx [4(2l-1)]^{-1}, \quad \text{if } l \to \infty.$$

Consequently, when an increase in Θ_a viscoplastic flow becomes purely viscous with $\bar{\eta}_0 \leqslant \eta_0 \leqslant 1$ ($\Delta\Theta_a < 0$), or changes into the two-layer mode of flow 1). When $3/4 < l < 1$ the reverse transition $P \to G$ occurs with a decrease in the temperature Θ_a from a high-temperature region, i.e., the flow changes from two-layer 1) or three-layer 2) modes and becomes viscoplastic.

For $l > 1$ (Fig. 1d, e and Fig. 2b) the curve $\Theta_a(\Theta_c)$ reaches a local minimum only in the two-layer mode of flow 1), so that discontinuous cooling takes place only from one mode of flow. The critical conditions for discontinuous heating and cooling do not coincide (a hysteresis effect).

The region of nonunique solutions, which lies between the curves $T_a^m(\tau_w)$ and $T_a^o(\tau_w)$ of critical temperatures for discontinuous transitions, is shown in Fig. 3. For larger values of the wall shear stresses the critical values asymptotically approach the parabola $\eta_w^2(1 - 16l/9)/4$, $l > 3/4$. With an increase in l (an increase in the channel width, improvement of the heat-exchange conditions, a decrease in the thermal conductivity of the wall) the region of nonunique solutions of the problem under study widens.

The influence of a change in the ambient temperature on the fluid flow rate (the pressure gradient is constant) is presented in Fig. 4. An increase in the temperature T_a to the critical value T_a^o does not affect the flow rate. At $T_a = T_a^o$ discontinuous heating of the fluid occurs owing to dissipative heat release and the flow rate abruptly grows.

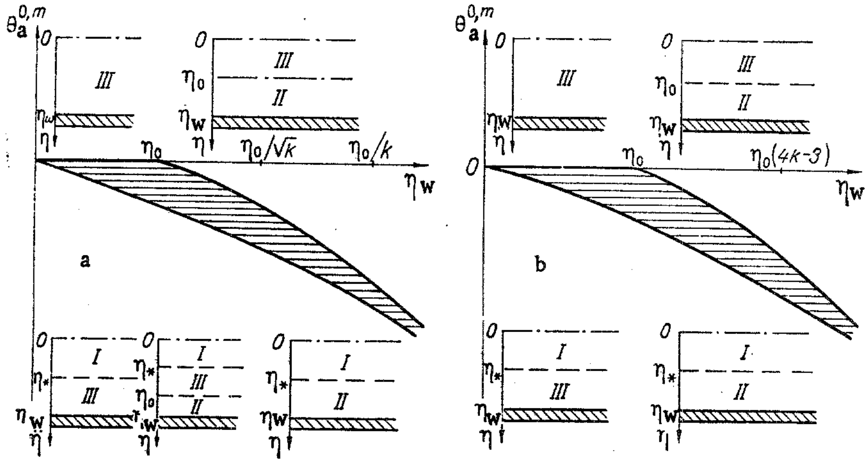

Fig. 3. Dependence of critical temperatures of discontinuous transitions on wall shear stress: a) $k < 1$; b) $k > 1$.

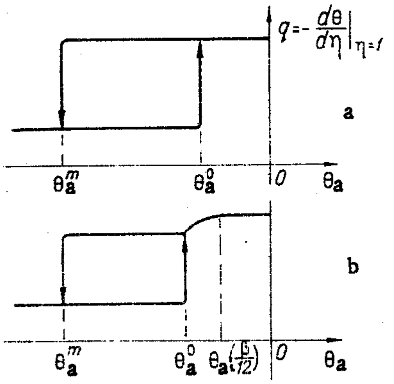

Fig. 4. Variation of dimensionless fluid flow rate $q = Q\tau_w/\lambda_f T_v$ with the ambient temperature Θ_a: a) $\eta_0 \geqslant \bar{\eta}_0$; b) $\eta_0 < \bar{\eta}_0$.

If $\eta_0 < \bar{\eta}_0$ then with a further increase in T_a the flow rate increases to the value corresponding to the flow of a purely viscous fluid. When $\eta_0 > \bar{\eta}_0$ this value of the flow rate is reached immediately after the discontinuous heating. Upon a decrease in the temperature T_a the flow rate abruptly decreases to the value corresponding to the flow of a viscoplastic fluid. The dependence of the flow rate on the ambient temperature also has a hysteresis character.

2. Flow of a fluid obeying the rheological equation of state

$$\varphi(\tau, T) = \begin{cases} k_1^{-\frac{1}{n}} \tau^{\frac{1-n}{n}} & T \leqslant T_p \\ k_2^{-\frac{1}{n}} \tau^{\frac{1-n}{n}} & T > T_p \end{cases}$$

in a plane channel with allowance for the dissipation of mechanical energy. By analogy with the preceding case we introduce the dimensionless shear stress η_* corresponding to the temperature $\Theta(\eta_*) = 0$ at which a sharp change in fluidity occurs.

We obtain the following sequences of zones of flow of a "power-law" fluid with coefficients of consistency k_2 $(0 \leqslant \eta \leqslant \eta_*)$ and k_1 $(\eta_* < \eta \leqslant 1)$ if $\Theta_c > 0$ and $0 \leqslant \eta_* \leqslant 1$; k_2 if $\Theta_c > 0$ and $\eta_* \geqslant 1$; k_1 if $\Theta_c < 0$.

From the solution of the problem (1) for the medium under consideration we have

$$\Theta_a = \left| \begin{array}{l} \beta_1 \dfrac{n}{2n+1}\left(\sigma - \dfrac{2n+1}{3n+1}\right)\eta_*^{\frac{3n+1}{n}} + \\ + \beta_1 \dfrac{n}{2n+1} l_1 [(1-\sigma)\eta_*^{\frac{2n+1}{n}} - 1] + \beta_1 \dfrac{n}{3n+1}, \end{array} \right.$$

$$\Theta_a = \begin{cases} \Theta_c = \beta_1\sigma \dfrac{n}{2n+1}\dfrac{n}{3n+1}\eta_*^{\frac{3n+1}{n}} & \Theta_c > 0,\ 0 \leqslant \eta_* < 1; \\ \Theta_c + \beta_1\sigma\dfrac{n}{3n+1} - l_1\beta_1\sigma\dfrac{n}{2n+1} & \Theta_c > 0,\ \eta_* \geqslant 1; \\ \Theta_c + \beta_1\dfrac{n}{3n+1} - l_1\beta_1\dfrac{n}{2n+1} & \Theta_c < 0. \end{cases}$$

Here $\sigma = (k_1/k_2)^{1/n}$, $l_1 = \dfrac{4}{3}l$, $\beta_1 = \dfrac{h^2\tau_w^{(n+1)/n}}{\lambda_f T_p k_1^{1/n}}$.

From an analysis of the function $\Theta_a(\Theta_c)$ it follows that when $\Theta_a \leqslant \Theta_a^m$ and $\Theta_a \geqslant \Theta_a^o$ the boundary problem (1) has a unique solution and when $\Theta_a^m < \Theta_a < \Theta_a^o$ it has three solutions, with one of them $(\partial\Theta_a/\partial\Theta_c < 0)$ being unstable. Let us analyze the difference between the values of the function $\Theta_a(\Theta_c)$ at $\Theta_c = 0$ and $\Theta_c = \beta_1\sigma \times [n/(2n+1)][n/(3n+1)]$ $(\eta_* = 1)$:

$$\Delta\Theta_a = \Theta_a|_{\eta_*=1} - \Theta_a|_{\eta_*=0} = \frac{n}{2n+1}\beta_1\left[\sigma(1-l_1) + l_1 - \frac{2n+1}{3n+1}\right].$$

Consequently, if $\Delta\Theta_a > 0$ then $1 < l < [\sigma - (2n+1)/(3n+1)]/(\sigma - 1)$ and the flow changes into a two-zone mode after the discontinuous heating. If $\Delta\Theta_a < 0$ then $[\sigma - (2n+1)/(3n+1)]/(\sigma - 1) < l < [(3n+1)/(2n+1)\,\sigma - 1]/(\sigma - 1)$, and after the heating we obtain the flow of a "power-law" fluid with a coefficient of consistency k_2.

For a numerical investigation of the problem (1) we chose an exponential law of variation of the plastic viscosity with temperature and a hyperbolic temperature dependence of the yield point for a Shvedov–Bingham viscoplastic medium. The calculations confirm the qualitative results obtained in the present report.

NOTATION

T_f, T_w, temperatures in fluid stream and at walls, respectively; y, transverse coordinate; τ, shear sterss; λ_f, λ_w, coefficients of thermal conductivity of fluid and walls, respectively; $\varphi(\tau, T)$, fluidity function; h, half-width of channel; b, wall thickness; τ_w, wall shear stress; τ_0, limiting shear stress of a viscoplastic fluid; α, coefficient of heat exchange; μ, characteristic viscosity of medium; T_a, ambient temperature; T_a^o, T_a^m, critical values of ambient temperature during heating and cooling, respectively; T_v, T_p, characteristic temperatures of a change in the rheological properties of the medium for viscoplastic and "power-law" fluids, respectively; Φ_Σ, total dissipation of mechanical energy in fluid stream; k_1, k_2, coefficients of consistency of a power-law fluid; Θ, η, $\varphi(\Theta, \eta)$, dimensionless temperature, stress, and fluidity function, respectively; Bi, Biot number; $\lambda = \lambda_f/\lambda_w$; $\sigma = (k_1/k_2)^{1/n}$; $l = \frac{3}{4}[1 + \lambda(\delta + 1/\mathrm{Bi})]$; $l_1 = 4l/3$; $\beta = h^2\tau_w^2/\lambda_f T_v \mu$; $\beta_1 = h^2\tau_w^{n+1/n}/\lambda_f T_v k_1$; δ, dimensionless wall thickness; η_w, dimensionless wall shear stress; η_0, dimensionless yield point; η_*, dimensionless stress corresponding to a discontinuous change in the properties of the medium; η_0, solution of the equation $\Delta\Theta_a = 0$; Θ_a, dimensionless ambient temperature; Θ_a^o, Θ_a^m, critical values of dimensionless ambient temperature during heating and cooling; Θ_w, dimensionless temperature of inner surface of channel wall; z, ξ, integration variables; $\Delta\Theta_a = \Theta_a|_{\eta_*=0} - \Theta_a|_{\eta_*=1}$; Θ_c, dimensionless temperature of midplane of channel.

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